Drill Problems 2
WITH ANSWERS

PROBLEM 1: *Bond Rating 1*

A triple A rated firm has a default probability of 0.0002 in any given year. What are the chances a triple A firm defaults in the next 25 years?

Answer: Pr(default in 25 yrs) = 1 − Pr(no default in 25 years)^{25} = 0.004988 .

PROBLEM 2: *Bond Rating 2*

A firm with a BBB bond rating has a 10.29 percent chance of default at some point over 40 years according to historical data. If the average BBB rated firm is in existence for 40 years, what is their default probability in any given year. Assume the chances from year to year are independent.

Answer:

   Pr(no default in 40 years) = 1 − Pr(default at some point) = 1 − 0.1029 = 0.8971 .
   Pr(no default in one year)^{40} = Pr(no default in 40 years) = 0.8971 .
   Pr(no default in one year) = (0.8971)$^{1/40}$ = 0.9973 .
   Therefore Pr(default with one year) = 1 − 0.9973 = 0.0027 .

PROBLEM 3: *Exercise on Mean and Variance*

Consider two random variables, X and Y, with the following distributions:

X	Pr(X=x)		y	Pr(Y=y)
3	20%		2	15%
6	22%		4	30%
10	58%		7	35%
			12	20%

1.   Assuming X and Y are independent, compute E[X] and E[Y].
2.   Assuming that X and Y are independent, compute both the variance and standard deviation of X + Y .

Answer:

   1. E(X) = 3 · 0.20 + 6 · 0.22 + 10 · 0.58 = 7.72 .
      E(Y) = 2 · 0.15 + 4 · 0.30 + 7 · 0.35 + 12 · 0.20 = 6.35

   2. Var(X) = 8.12, and Var(Y) = 11.03 .
      Since X and Y are independent, Var(X + Y) = Var(X) + Var(Y) ,
      Var(X + Y) = Var(X) + Var(Y) = 19.15 , and

$$\text{StDev}(X+Y)=\sqrt{\text{Var}(X+Y)}=\sqrt{19.15}=4.38$$

PROBLEM 4: *Stock Return*

Shares in companies A, B, and C have expected rates of return over the next year of 10%, 11%, and 15%, respectively. The standard deviation of the rate of return of each of the stocks is 6%, 8%, and 10% respectively. The returns on the stocks vary independently.

1. What is the expected value and standard deviation of the rate of return (over the next year) on a portfolio consisting of equal dollar amounts invested in all three stocks?

2. What if the portfolio puts 20% of your investment in A, 30% in B, and 50% in C?

Answer:

1. Denote by R_A, R_B, and R_C the return on the stocks A, B, and C respectively.
The return on a portfolio with equal amounts of each stock is the random variable
$X = (R_A + R_B + R_C) / 3$.

$$E[X] = E[\ (R_A + R_B + R_C)/3] = (0.1 + 0.11 + 0.15) / 3 = 0.12$$

Since the returns are independent,

$$Var(X) = Var(R_A/3) + Var(R_B/3) + Var(R_C/3) = 0.06^2/9 + 0.08^2/9 + 0.1^2/9 = 0.0022$$

Therefore the standard deviation of the returns is $\sqrt{0.0022} = 0.047$.

2. Let $Y = 0.2 \cdot R_A + 0.3 \cdot R_B + 0.5 \cdot R_C$. Then

$$E[Y] = 0.2 \cdot R_A + 0.3 \cdot R_B + 0.5 \cdot R_C = 0.2 \cdot 0.10 + 0.3 \cdot 0.11 + 0.5 \cdot 0.15 = 0.128 = 12.8\%,$$

and (due to independence)

$$Var(Y) = Var(0.2 \cdot R_A) + Var(0.3 \cdot R_B) + Var(0.5 \cdot R_C)$$

$$= 0.2^2 \cdot Var(R_A) + 0.3^2 \cdot Var(R_B) + 0.5^2 \cdot Var(R_C)$$

$$= 0.2^2 \cdot 0.06^2 + 0.3^2 \cdot 0.08^2 + 0.5^2 \cdot 0.1^2$$

$$= 0.00322, \text{ and}$$

$$StDev(Y) = 0.0567 = 5.67\%.$$